Math 115

Spring 2019

Lecture 5

$?\ a^2 + b^2 = c^2\ ?$
$y = mx + b$ $?$ $d = rt$

Feb 19-8:47 AM

Some Review:

1) Evaluate $\quad -b - \sqrt{b^2 - 4ac} \qquad$ for $a = 2$, $b = 10$,

$\qquad\qquad$ and $c = 8$.

$= -10 - \sqrt{10^2 - 4(2)(8)}$

$= -10 - \sqrt{100 - 64} = -10 - \sqrt{36} = -10 - 6 = \boxed{-16}$

2) Simplify: $\quad 3\left(\frac{1}{3}x^2 - 2x + 1\right) + 6x - 3$

$= 3\cdot\left(\frac{1}{3}x^2\right) - 3(2x) + 3\cdot 1 + 6x - 3$

$= \left(3\cdot\frac{1}{3}\right)x^2 - (3\cdot 2)x + 3\cdot 1 + 6x - 3$

$= 1x^2 - 6x + 3\cdot 1 + 6x - 3 = x^2 - 6x + 3 + 6x - 3 = \boxed{x^2}$

3) Solve: $\quad 2(3x - 1) + 7 = x - 25$

$\qquad\qquad 6x - 2 + 7 = x - 25$

$\qquad\qquad 6x + 5 = x - 25$

$\qquad\qquad 6x - x = -25 - 5$

$\qquad\qquad 5x = -30$

$\qquad\qquad\qquad\qquad x = \dfrac{-30}{5}$

$\qquad\qquad\qquad\qquad \boxed{x = -6}$

$\qquad\qquad\qquad\qquad \{-6\}$
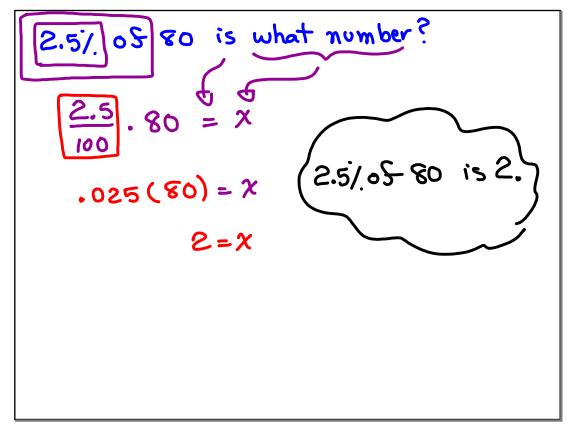
4) Simplify: $\dfrac{\sqrt{121}-|-10|}{\sqrt{5^2-(-4)^2}-(-2)^2} = \dfrac{11-10}{\sqrt{25-16}-4} = \dfrac{1}{\sqrt{9}-4}$

$= \dfrac{1}{3-4} = \dfrac{1}{-1} = \boxed{-1}$

5) Simplify: $-2\left(3x^2+5x+\tfrac{1}{2}\right)+6\left(x^2-2x+\tfrac{1}{6}\right)$

$= -2(3x^2)-2(5x)-2\cdot\tfrac{1}{2}+6x^2-6(2x)+6\cdot\tfrac{1}{6}$

$= -6x^2-10x-1+6x^2-12x+1 = -10x-12x = \boxed{-22x}$

6) Solve $5(2x-7)+13 = 2(5x+4)-8$

$10x-35+13 = 10x+8-8$

$10x-22 = 10x$

$10x-10x = 22$

$0 = 22$ → False

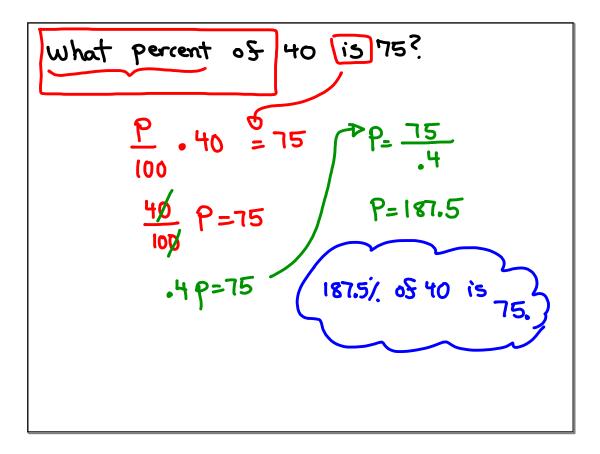$\boxed{\phi}$ No Soln.

7) Solve $\tfrac{1}{4}x-\tfrac{1}{2} = \tfrac{1}{2}x+\tfrac{3}{4}$ LCD=4

$4\cdot\tfrac{1}{4}x-\overset{2}{4}\cdot\tfrac{1}{2} = \overset{2}{4}\cdot\tfrac{1}{2}x+4\cdot\tfrac{3}{4}$

$x-2 = 2x+3$

$x-2x = 3+2$

$-x = 5$

$x = \dfrac{5}{-1}$ $\boxed{x=-5}$ $\{-5\}$

8) Is 5 a Solution of $\sqrt{x-1}+\sqrt{x+4} = -5$?

$\sqrt{5-1}+\sqrt{5+4} \overset{?}{=} -5$

$\sqrt{4}+\sqrt{9} \overset{?}{=} -5$

$2+3 \overset{?}{=} -5$

$5 \overset{?}{=} -5$ false

Not a Soln.

9) Is -5 a Solution of $|2x-1| = 11$?

$|2(-5)-1| \overset{?}{=} 11$

$|-10-1| \overset{?}{=} 11$

$|-11| \overset{?}{=} 11$
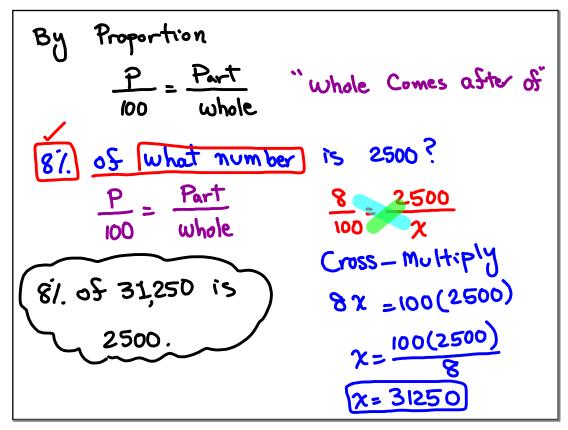
$11 = 11$ True

-5 is a Soln.

10) Translate: 4 times some number increased by -5.

Let x be the number,

$$4x + (-5) = 4x - 5$$

11) Translate: 4 times the difference of

Some number and 5 is equal to 10

less than the number

Let x be the number

$$4(x - 5) = x - 10$$

Basic Percent

By Translation

what, what number x

P%, what percent $\frac{P}{100}$

% of $\cdot$

is, get, result, ... $=$

4% of what number is 160?

$$\frac{4}{100} \cdot x = 160$$

$$.04x = 160$$

$$x = \frac{160}{.04} \qquad x = 4000$$

4% of 4000 is 160.

2.5% of 80 is what number?

$\dfrac{2.5}{100} \cdot 80 = x$

$.025(80) = x$

$2 = x$

2.5% of 80 is 2.

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what percent of 40 is 75?

$\dfrac{P}{100} \cdot 40 = 75$

$\dfrac{40}{100} P = 75$

$.4P = 75$

$P = \dfrac{75}{.4}$

$P = 187.5$

187.5% of 40 is 75.

By Proportion

$$\frac{P}{100} = \frac{Part}{whole}$$

"Whole comes after of"

✓ 8% of what number is 2500?

$$\frac{P}{100} = \frac{Part}{whole}$$

$$\frac{8}{100} = \frac{2500}{x}$$

Cross-Multiply

$$8x = 100(2500)$$

$$x = \frac{100(2500)}{8}$$
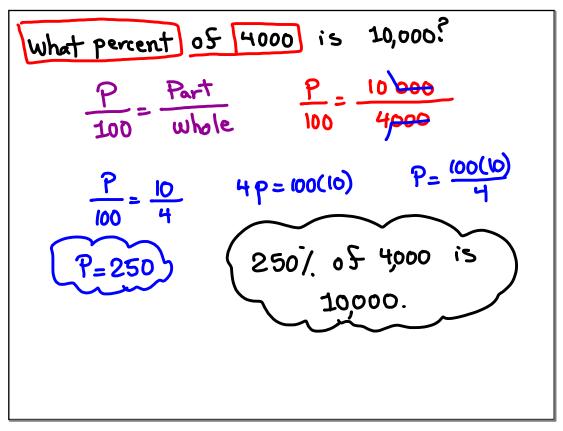
$$\boxed{x = 31250}$$

8% of 31,250 is 2500.

1.25% of 16000 is what number?

$$\frac{P}{100} = \frac{Part}{whole}$$

$$\frac{1.25}{100} = \frac{x}{16000}$$

Cross Multiply

$$100x = 16000(1.25)$$

$$x = \frac{16000(1.25)}{100}$$

$$x = 160(1.25)$$

$$\boxed{x = 200}$$

1.25% of 16,000 is 200.

What percent of 4000 is 10,000?

$$\frac{P}{100} = \frac{Part}{whole} \qquad \frac{P}{100} = \frac{10\,000}{4000}$$

$$\frac{P}{100} = \frac{10}{4} \qquad 4P = 100(10) \qquad P = \frac{100(10)}{4}$$

$$P = 250$$

250% of 4,000 is 10,000.

Types of linear equations:

1) Conditional: when there is exactly one Solution.

2) Contradiction: when there is no Soln.

3) Identity: when there are infinitely many Solutions.

Solve, and identify the type of equation

$3(x-2) - (x+4) = x + 10$

$3x \; -6 \; -x \; -4 = x + 10$

$2x \; -10 = x + 10$

$2x - x = 10 + 10$

$\boxed{x = 20} \rightarrow \{20\}$

Since there is exactly one Solution, Equation is Conditional.

Solve, then identify the type of equation.

$3\left(\frac{1}{3}x - 4\right) + 8 = \frac{1}{4}(4x - 8) + 2$

$3 \cdot \frac{1}{3}x - 3 \cdot 4 + 8 = \frac{1}{4} \cdot 4x - \frac{1}{4} \cdot 8^2 + 2$

$x - 12 + 8 = x - 2 + 2$

$x - 4 = x$

$x - x = 4$ $\rightarrow$ false

$0 = 4$ $\rightarrow$ NO Solution $\Rightarrow \boxed{\phi}$

$\Rightarrow \{ \; \}$

Since there is no Solution, equation is a Contradiction.

Solve, then identify the type of equation!

$$5\left(\frac{2}{5}x - 1\right) - \frac{1}{4}(4x - 4) = 6\left(\frac{1}{6}x + 1\right) - 10$$

$$5 \cdot \frac{2}{5}x - 5 \cdot 1 - \frac{1}{4} \cdot 4x - \frac{1}{4}(-4) = 6 \cdot \frac{1}{6}x + 6 \cdot 1 - 10$$

$$2x \; \boxed{-5} \; -x \; \boxed{+1} \; = x \; \boxed{+6} \; \boxed{-10}$$

$$x \; -4 = x - 4$$

$$x - x = -4 + 4$$

$$0 = 0$$

True

→ infinitely many

Solutions ⟹ $\mathbb{R}$

All Real numbers

Since there are infinitely many Solns, equation is an identity

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Cross – Multiply, then Solve

$$\frac{2x + 5}{2} = \frac{x - 4}{5}$$

$$\left\{\frac{-33}{8}\right\}$$

Equation is Conditional

$$5(2x + 5) = 2(x - 4)$$

$$10x \; \boxed{+25} = \boxed{2x} - 8$$

$$10x - 2x = -8 - 25$$

$$8x = -33$$

$$\boxed{x = \frac{-33}{8}}$$

$$x = -4\frac{1}{8} \qquad x = -4.125$$

Cross-Multiply, and Solve

$$\frac{3x - 5}{3} = \frac{2x + 4}{2}$$

$$2(3x - 5) = 3(2x + 4)$$

$$6x - 10 = 6x + 12$$

$$6x - 6x = 12 + 10$$

$$0 = 22$$

⊳ False

⇒ No Solution

$\emptyset$

Eqn is Contradiction.

Solve $\quad \dfrac{2x - 4}{4} = \dfrac{x - 2}{2}$

Cross-Multiply

$$2(2x - 4) = 4(x - 2)$$

$$4x - 8 = 4x - 8$$

$$4x - 4x = -8 + 8$$

$$0 = 0$$

⊳ True

infinite number of Solutions

$\mathbb{R}$

All Real numbers.

Equation is an identity.

Simplify $\quad \dfrac{2}{3} - \dfrac{1}{4} \div \dfrac{3}{5}$

$$= \dfrac{2}{3} - \dfrac{1}{4} \cdot \dfrac{5}{3}$$

$$= \dfrac{2}{3} - \dfrac{5}{12}$$

LCD

$$3 = 3 \; \square \; \square$$

$$12 = 3 \cdot 2 \cdot 2$$

$$\overline{LCD = 3 \cdot 2 \cdot 2 = 12}$$

$$= \dfrac{2 \cdot 4}{3 \cdot 4} - \dfrac{5}{12}$$

$$= \dfrac{8}{12} - \dfrac{5}{12} = \dfrac{3}{12} = \dfrac{3 \cdot 1}{3 \cdot 4} = \boxed{\dfrac{1}{4}}$$

Simplify $\quad \left(\dfrac{3}{5} - \dfrac{1}{4} \right) \div \left(\sqrt{\dfrac{1}{16}} - \sqrt{\dfrac{9}{25}} \right)$

$$= \left(\dfrac{3}{5} - \dfrac{1}{4} \right) \div \left(\dfrac{1}{4} - \dfrac{3}{5} \right)$$

$$= \dfrac{\dfrac{3}{5} - \dfrac{1}{4}}{\dfrac{1}{4} - \dfrac{3}{5}}$$

Complex fraction

To Simplify

Multiply everything

by LCD = 20

$$= \dfrac{20 \cdot \dfrac{3}{5} - 20 \cdot \dfrac{1}{4}}{20 \cdot \dfrac{1}{4} - 20 \cdot \dfrac{3}{5}} = \dfrac{12 - 5}{5 - 12} = \dfrac{7}{-7} = \boxed{-1}$$

Evaluate $\dfrac{\dfrac{1}{x} - \dfrac{1}{y}}{\dfrac{x}{z}}$ for $x=5$, $y=2$, $z=6$.

$\dfrac{\dfrac{1}{5} - \dfrac{1}{2}}{\dfrac{5}{6}} = \dfrac{\overset{6}{\cancel{30}} \cdot \dfrac{1}{\cancel{5}} - \overset{15}{\cancel{30}} \cdot \dfrac{1}{\cancel{2}}}{\cancel{5} \,\, \cancel{30} \cdot \dfrac{5}{\cancel{6}}} = \dfrac{6 - 15}{25}$

$= \boxed{\dfrac{-9}{25}}$

LCD = 30

$5 = 5$

$2 = 2$

$6 = 2 \cdot 3$

$\overline{\text{LCD} = 5 \cdot 2 \cdot 3 = 30}$

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4 more than   3 times some number

is equal to

The difference of the number and 10.

find square of the number. Let x be the number,

$3x + 4 = x - 10$

$3x - x = -10 - 4$

$2x = -14$

$(-7)^2 = 49$

$x = \dfrac{-14}{2}$ $\boxed{x = -7}$

the number is -7.

Square of the number is 49

half the sum of some number and 5

is equal to

10 less the number. find the number.

Let x be the number,

$$\frac{1}{2}(x+5) \qquad = \qquad 10 - x$$

$$\frac{1}{2}(x+5) = 10 - x$$

LCD = 2

$$2 \cdot \frac{1}{2}(x+5) = 2 \cdot 10 - 2 \cdot x$$

$$x + 5 = 20 - 2x$$

$$x + 2x = 20 - 5$$

$$3x = 15$$

$$x = \frac{15}{3} \qquad \boxed{x = 5}$$

the number is 5.